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Gauge Dependence of the High-Temperature 2-Loop Effective Potential for the Higgs Field

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Abstract

The high-temperature limit of the 2-loop effective potential for the Higgs field is calculated from an effective 3d theory, in a general covariant gauge. It is shown explicitly that a gauge-independent result can be extracted for the equation of state from the gauge-dependent effective potential. The convergence of perturbation theory is estimated in the broken phase, utilizing the gauge dependence of the effective potential.

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1 Introduction

Recently, the high-temperature electroweak phase transition has been the subject of active research, due to its possible effect on the baryon number of the Universe. The standard tool for perturbative investigations of the equilibrium properties of this phase transition is the effective potential for the Higgs field, $V(\varphi)$. From the effective potential, one can calculate quantities like the order, the critical temperature, and the latent heat of the phase transition. In particular, the pressure is given by minus the value of the effective potential at the minimum. Presently, the most complete expressions for $V(\varphi)$ are given by the 2-loop calculations in refs. [1, 2, 3].

When calculating the effective potential, one of the problems one is faced with is gauge dependence. Indeed, the source term appearing in the generating functional

$$\exp(-W[J]) \equiv \int \mathcal{D}\phi \mathcal{D}A \exp(-S[\phi, A] - (J^\dagger \Phi + \Phi^\dagger J)/2) \tag{1}$$

is not gauge-invariant, and hence $W[J]$ may be gauge-dependent. The effective potential $V(\varphi)$, which is obtained from the Legendre transformation of $W[J]$, then also depends on the gauge condition [4]. In particular, the location of the minimum of the effective potential is gauge-dependent, which indicates that φ is not a physical observable. However, the value of the effective potential at a stationary point is obtained by putting the source J to zero, and consequently one should obtain a gauge-independent result for the pressure [5, 6]. It is the main purpose of this paper to check explicitly to 2-loop order ($\hbar^2$) that a gauge-independent result is, indeed, obtained.

There are methods of extending the definition of the effective potential so that it is gauge-independent even away from the minima (see, e.g., ref. [7]). A concrete example, relevant for the EW phase transition, was given in ref. [8]. There the external source was coupled to the composite operator $\Phi^\dagger\Phi$:

$$\exp(-\widetilde{W}[J]) \equiv \int \mathcal{D}\phi \mathcal{D}A \exp(-S[\phi, A] - 2J\Phi^\dagger \Phi) \ . \tag{2}$$

The source term $2J\Phi^\dagger\Phi$, and hence both $\widetilde{W}[J]$ and the corresponding effective potential $\widetilde{V}(\sigma)$, are manifestly gauge-independent. The pressure is again obtained from the value of $\widetilde{V}(\sigma)$ at its minimum, or, equivalently, from $\widetilde{W}[0]$. It is checked below to order $\hbar^2$ that $\widetilde{V}(\sigma)$ and the conventional gauge-dependent effective potential $V(\varphi)$ give the same result for the pressure.

The paper is organized as follows. In Sec. 2Method of calculationsection.2, some technical details of the calculation of the 2-loop effective potential in a general covariant gauge are discussed. In Sec. 3Equation of statesection.3, it is shown that the value of the effective potential at the minimum is gauge-independent to order $\hbar^2$. In Sec. 4Gauge-independent effective potentialsection.4, it is shown that the effective potentials resulting from eqs. 1 and 2 yield the same values for the pressure, when calculated consistently in powers of $\hbar$. The calculation of some other physical quantities

than pressure is discussed in Sec. 5Other physical quantities in powers of $\hbar$section.5. In Sec. 6Convergence in the broken phasesection.6, the convergence of perturbation theory is studied in the broken phase, utilizing the gauge-dependent effective potential. The conclusions are in Sec. 7Conclusionssection.7. The explicit form of the 2-loop effective potential is represented in the Appendix. Sections 2Method of calculationsection.2 and 6Convergence in the broken phasesection.6 are an extension of ref. [9].

2 Method of calculation

In the high-temperature limit, the essential long-wavelength properties of the Standard Model can be described by an effective super-renormalizable 3-dimensional field theory [10, 11]. This dimensional reduction is accomplished by integrating out all the degrees of freedom corresponding to the momentum scale $p \sim T$ in the original 4d theory. At the 2-loop level, it has been explicitly verified [3] that the 3d theory produces the high-temperature limit of the 4d effective potential for the Higgs field. Furthermore, the parameters of the effective 3d theory should be independent of the 4d gauge fixing condition [3, 12]. The relevant 3d action is

$$S = \int d^3x \Big[\frac{1}{4} F^a_{ij} F^a_{ij} + (D_i\Phi)^\dagger (D_i\Phi) + m_3^2 \Phi^\dagger\Phi + \lambda_3 (\Phi^\dagger\Phi)^2 + h_3 \Phi^\dagger\Phi A^a_0 A^a_0 \\ + \frac{1}{2}(D_i A^a_0)^2 + \frac{1}{2} m_D^2 A^a_0 A^a_0 + \frac{1}{4}\lambda_A (A^a_0 A^a_0)^2 \Big], \tag{3}$$

where $F^a_{ij} = \partial_i A^a_j - \partial_j A^a_i + g_3 \epsilon^{abc} A^b_i A^c_j$, $D_i\Phi = (\partial_i - i g_3 \tau^a A^a_i/2)\Phi$, and $D_i A^a_0 = \partial_i A^a_0 + g_3 \epsilon^{abc} A^b_i A^c_0$. The τ^a:s are the Pauli matrices. All the fields have the dimension GeV$^{1/2}$, and λ_3, g_3^2, h_3 and λ_A have the dimension GeV.

The theory in eq. 3 is a simplified theory in the sense that the U(1)-subgroup has been neglected. All the fermions can be included, but they appear only in the parameters of the effective theory. The relations of the parameters of the 3d theory to those of the Standard Model are given in refs. [3, 13], but we do not presently need these relations. In some cases, the 3d theory could be further simplified by integrating out also the A^a_0-field [3, 14], resulting in a 3d SU(2)-Higgs model. For generality, we keep the A^a_0-field in the action. In the following, we calculate the effective potential in the theory defined by eq. 3. Due to the change in the dimension of the space in the reduction step, the value of this effective potential at the minimum is then minus the pressure divided by the temperature.

The Lagrangian in eq. 3 is gauge-invariant in τ-independent gauge transformations, and gauge fixing and compensating terms have to be added for perturbative calculations. We choose these as

$$S_\xi = \int d^3x \Big[\frac{1}{2\xi} (\partial_i A^a_i)^2 + \partial_i \bar{c}^a \partial_i c^a + g_3 \epsilon^{abc} \partial_i \bar{c}^a A^b_i c^c \Big] . \tag{4}$$

The gauge parameter ξ is a renormalized version of the corresponding 4d gauge fixing parameter. The covariant gauge condition of eq. 4 has both advantages and disadvantages compared with other usual gauges, like the R_ξ-gauge. On one hand, it is clearly an allowed choice of gauge, whereas with the R_ξ-gauge, one is choosing a different gauge for each different value of φ in the effective potential, which might not be allowed. On the other hand, the covariant gauge generates extra IR-divergences, as is seen below.

To calculate the effective potential $V(\varphi)$, one writes $\Phi = [\phi_3+i\phi_4, \varphi+\phi_1+i\phi_2]^T/\sqrt{2}$ in the action $S+S_\xi$ and neglects terms linear in quantum fields [4]. This defines a new theory with the masses $m_1^2 \equiv m_3^2 + 3\lambda_3\varphi^2$, $m_2^2 \equiv m_3^2 + \lambda_3\varphi^2$, $m_L^2 \equiv m_D^2 + h_3\varphi^2$, and $m_T^2 \equiv g_3^2\varphi^2/4$. The propagators are

$$
\begin{aligned}
\underbrace{\overline{c}^a(k)c^b}(k) &= \frac{\delta^{ab}}{k^2} \\
\underbrace{\phi_1(-k)\phi_1}(k) &= \frac{1}{k^2+m_1^2} \\
\underbrace{A_0^a(-k)A_0^b}(k) &= \frac{\delta^{ab}}{k^2+m_L^2} \\
\underbrace{A_i^a(-k)A_j^b}(k) &= \delta^{ab}\left[\frac{\delta_{ij}-k_ik_j/k^2}{k^2+m_T^2} + \xi\frac{k_ik_j}{k^2}\frac{k^2+m_2^2}{k^2(k^2+m_2^2)+\xi m_T^2 m_2^2}\right] \qquad (5) \\
\underbrace{\phi_G(-k)\phi_G}(k) &= \frac{k^2+\xi m_T^2}{k^2(k^2+m_2^2)+\xi m_T^2 m_2^2}\,, \qquad G=2,3,4 \\
\underbrace{\phi_2(k)A_i^3}(-k) &= -\underbrace{\phi_3(k)A_i^2}(-k) = -\underbrace{\phi_4(k)A_i^1}(-k) = \frac{i\xi m_T k_i}{k^2(k^2+m_2^2)+\xi m_T^2 m_2^2}\,.
\end{aligned}
$$

If m_2^2 is negative, a small imaginary part has to be added to it to define the loop integrals. One then calculates all the one-particle-irreducible vacuum diagrams of the theory to a desired order in the loop expansion. Non-vanishing 2-loop contributions arise from the diagrams of Fig. 1. The method of calculation is to write

$$
\frac{1}{k^2(k^2+m_2^2)+\xi m_T^2 m_2^2} = \frac{1}{m_2^2(R_+^2-R_-^2)}\left[\frac{1}{k^2+m_2^2R_-^2} - \frac{1}{k^2+m_2^2R_+^2}\right]\,, \qquad (6)
$$

where $R_\pm^2 = 1/2 \pm \sqrt{1/4-\xi(m_T/m_2)^2}$, and to use standard Landau-gauge values of integrals [3], with dimensional regularization. The 2-loop effective potential is presented in the Appendix.

Next, the effective potential has to be renormalized. Many of the individual 2-loop diagrams contributing to $V(\varphi)$ include gauge-dependent divergent pieces, but all these cancel, leaving the gauge-independent divergence

$$
\frac{\hbar^2}{16\pi^2}\frac{\mu^{-4\epsilon}}{4\epsilon}\left\{\frac{\varphi^2}{2}\left[\frac{39}{16}g_3^4 + 9\lambda_3 g_3^2 - 12\lambda_3^2 + 12h_3g_3^2 - 6h_3^2\right] + 3g_3^2m_3^2 + 6g_3^2m_D^2\right\}\,. \qquad (7)
$$

The piece multiplying φ^2 is removed by mass renormalization, and the coupling constants are RG-invariant. Note that if the vacuum terms are renormalized by just

removing the $1/\epsilon$-pieces, the value of the effective potential at the minimum becomes μ-dependent. The φ-dependent part of $V(\varphi)$ is μ-independent, since the μ-dependence of the renormalized mass squared $m_3^2(\mu)$ cancels the $\log(\mu)$-terms of the 2-loop graphs. The renormalized effective potential is obtained from the expressions in the Appendix by replacing m_3^2 with $m_3^2(\mu)$, and by ignoring the $1/\epsilon$-piece in the function $H(m_a, m_b, m_c)$.

3 Equation of state

To calculate the value of $V(\varphi)$ at the minimum in powers of $\hbar$, we write $V(\varphi)$ as $V = V_0 + \hbar V_1 + \hbar^2 V_2$, with the classical part $V_0 = m_3^2\varphi^2/2 + \lambda_3\varphi^4/4$. The location of the minimum is determined from

$$0 = \frac{d}{d\varphi}\left[V_0(\varphi)|_{\varphi=\varphi_0+\hbar\varphi_1+\hbar^2\varphi_2} + \hbar\, V_1(\varphi)|_{\varphi=\varphi_0+\hbar\varphi_1} + \hbar^2\, V_2(\varphi)|_{\varphi=\varphi_0}\right] . \tag{8}$$

The $\hbar^0$-term of this equation reads $V_0'(\varphi_0) = \varphi_0 m_2^2 = 0$, yielding the two solutions $\varphi_0 = 0$ and $\varphi_0^2 = -m_3^2/\lambda_3$. Let us assume that $m_3^2 < 0$, and inspect first the broken minimum. From eq. 8, the corrections to φ_0 are

$$\varphi_1 = -\frac{V_1'}{V_0''} \qquad ; \qquad \varphi_2 = -\frac{1}{V_0''}\frac{d}{d\varphi}\left[V_2 - \frac{1}{2}\frac{(V_1')^2}{V_0''}\right] , \tag{9}$$

where each expression is evaluated at φ_0. The value of the 3d effective potential at the minimum is then

$$-\frac{p(T)}{T} = V_0(\varphi_0) + \hbar\Big[V_1 + \varphi_1 V_0'\Big]_{\varphi=\varphi_0} + \hbar^2\left[V_2 - \frac{1}{2}\frac{(V_1')^2}{V_0''} + \varphi_2 V_0'\right]_{\varphi=\varphi_0} . \tag{10}$$

This should be gauge-independent.

At φ_0, $V_0' = 0$ by definition, so that the terms multiplying φ_1 and φ_2 in eq. 10 should be put to zero. However, these terms are kept for the moment, for the following reason. In the limit $m_2^2 \to 0$ the Goldstone mode propagator is

$$\frac{1}{k^2} + \frac{\xi m_T^2}{k^4} . \tag{11}$$

Because this is IR-divergent inside loop integrals, the 2-loop effective potential diverges in the limit $m_2^2 \to 0$. The divergences from the individual diagrams which are of the form $\xi^{5/2} m_2^{-1}$, $\xi^{7/4} m_2^{-1/2}$, or $\xi^2 \log(m_2)$, cancel, but the divergent piece

$$\frac{1}{16\pi^2}\left[\frac{9}{8}\lambda_3 m_T^2 \xi^{3/2}\left(\frac{m_T}{m_2}\right) - \frac{9}{2\sqrt{2}}\lambda_3 m_T \xi^{3/4}\left(\frac{m_T}{m_2}\right)^{1/2}\left(m_1 + m_T\frac{g_3^2}{2\lambda_3} + m_L\frac{h_3}{\lambda_3}\right)\right] \tag{12}$$

remains. These problems show up even at the 1-loop level: the ξ-dependent part of V_1 is finite but non-analytic,

$$V_1^{(\xi)} \propto -\xi^{3/4} m_2^{3/2} . \tag{13}$$

Due to the equation $dm_2^r/d\varphi = rm_2^{r-2}\lambda_3\varphi$, it is seen from eqs. 9, 12 and 13 that the correction φ_1 diverges as $-\xi^{3/4}m_2^{-1/2}$, and φ_2 might diverge as $\xi^{3/2}m_2^{-3}$. However, it can easily be seen from eq. 23 that the term

$$-\frac{1}{2}\frac{(V_1')^2}{V_0''} \tag{14}$$

exactly cancels the divergent piece of V_2, shown in eq. 12. Consequently, it turns out that φ_2 only diverges as $-\xi m_2^{-1}$, so that being multiplied by $V_0' \propto m_2^2$ in eq. 10, it does not contribute to the value of the effective potential at the minimum, where $m_2^2 = 0$. In other words, the terms $\varphi_1 V_0'$ and $\varphi_2 V_0'$ can safely be put to zero, when the IR-divergences are "regularized" by handling the classical minimum as the limit $m_2 \to 0^+$. Let us mention that in the Landau-gauge ($\xi = 0$), φ_1 and φ_2 are finite.

In addition to cancelling all the divergences, the term in eq. 14 also cancels the finite gauge-dependent piece of V_2 in eq. 10. As mentioned, the gauge dependence of V_1 is of the form $\xi^{3/4}m_2^{3/2}$ near φ_0, so that this vanishes, too. Thus, we have verified explicitly that the pressure at the broken minimum is gauge-independent to order $\hbar^2$.

At the symmetric minimum $\varphi = 0$, to 2-loop order there are no IR-divergences in V_2, and the effective potential is gauge-independent in itself. The "correction term" in eq. 14 vanishes. If $m_3^2 < 0$, the effective potential becomes complex, in accordance with the fact that the symmetric minimum is, at the tree level, unstable for $m_3^2 < 0$.

In addition to ξ, all the physical quantities should be independent of the renormalization scale μ. As mentioned after eq. 7, in dimensional regularization the dimensionally reduced effective theory produces a μ-dependent unphysical vacuum term to the 3d effective potential, corresponding to a zero-temperature entropy. This term is actually removed by a counterterm produced by the dimensional reduction step, but on the other hand there is an undetermined vacuum energy density in the full 4d theory. Apart from the vacuum term, eq. 10 is μ-independent to order $\hbar^2$, since the φ-dependent part of $V(\varphi)$ is so. In particular, the difference between the pressures of the symmetric and broken phases, determining the critical temperature and the latent heat of the phase transition, is independent of both μ and ξ to order $\hbar^2$, as it should.

4 Gauge-independent effective potential

Let us calculate the gauge-independent generating functional defined in eq. 2, and the value of the corresponding effective potential at the minimum, in powers of $\hbar$ inside the 3d theory. Since we are only interested in the effective potential, J may be chosen as a constant. Defining $w(J) \equiv \widetilde{W}[J]/V$ and $dw/dJ = \sigma$, the value of the effective potential $\widetilde{V}(\sigma) = w - \sigma J$ at the minimum is formally

$$\widetilde{V}(\sigma)\Big|_{d\widetilde{V}/d\sigma=0} = w(0) \; . \tag{15}$$

To see how this equation arises in powers of $\hbar$, write $w(J) = w_0 + \hbar w_1 + \hbar^2 w_2$. For a Legendre transformation, the source $J = J_0 + \hbar J_1 + \hbar^2 J_2$ has to be solved from the equation $dw/dJ = \sigma$. With J_0 determined from $w_0'(J_0) = \sigma$, the effective potential is then

$$\widetilde{V}(\sigma) = w_0(J_0) - \sigma J_0 + \hbar w_1(J_0) + \hbar^2 \left[w_2 - \frac{1}{2}\frac{(w_1')^2}{w_0''} \right]_{J=J_0} . \tag{16}$$

Calculating the value of $\widetilde{V}(\sigma)$ at the minimum as in eqs. 8-10, finally gives the result $\widetilde{V}(\min) = w_0(0) + \hbar w_1(0) + \hbar^2 w_2(0)$ as in eq. 15. Below, we see explicitly that $w_0(0) + \hbar w_1(0) + \hbar^2 w_2(0)$ is given by eq. 10, so that as far as the equation of state is concerned, the conventional effective potential $V(\varphi)$ and the gauge-independent effective potential $\widetilde{V}(\sigma)$ should lead to the same physics.

To calculate $w(J)$ [and, in particular, $w(0)$], we need to perform the path integral

$$\int \mathcal{D}\phi \mathcal{D}A \exp(-S[\phi, A]) \tag{17}$$

using the loop expansion, and then to substitute $m_3^2 \to m_3^2 + 2J$. We first discuss the broken minimum. Let us make the change of variables $\Phi = (0, \varphi_0)^T/\sqrt{2} + \Phi'$ in eq. 17, where $\varphi_0 = \sqrt{-m_3^2/\lambda_3}$ is the location of the classical broken minimum. Since this is a stationary point of the Lagrangian, all terms linear in quantum fields disappear from the action (apart from counterterms contributing to higher order than $\hbar^2$). This means that we have exactly the same Lagrangian as in the case of calculating the effective potential $V(\varphi)$ at $\varphi = \varphi_0$. However, the set of diagrams is different, since in the calculation of the effective potential, only one-particle-irreducible graphs are included [4]. Now we have to include all the connected graphs, which means that one-particle-irreducible graphs have to be supplemented by graphs of the type shown in Fig. 2. These graphs can be easily calculated in the same gauge which was used before. The result is simple: these graphs contribute the amount $-(V_1')^2/(2V_0'')$. Hence, $w(0)$ equals the gauge-independent value of eq. 10, as promised.

When constructing the gauge-independent effective potential in the symmetric phase, there is the complication that the tree-level part of the generating function $w(J)$ vanishes, $w_0(J) = 0$. Therefore, the calculations after eq. 15 have to be modified. Nevertheless, it is still true that $w(0)$ as calculated from the loop expansion equals the value of the conventional effective potential at $\varphi = 0$.

5 Other physical quantities in powers of $\hbar$

Having constructed an expression for the value of the 3d effective potential at its minima, which is independent of ξ and independent of μ to order $\hbar^2$, we could express the 3d parameters in terms of the 4d parameters and temperature, and study the properties of the EW phase transition. Of course, it is expected that this investigation breaks down at some order, due to the IR-divergences of finite-temperature field theory.

Let us see, how far we can go without problems. From the difference $\Delta p(T)$ of the pressures of the symmetric and broken phases, we can in principle calculate the critical temperature T_c and the latent heat L to order $\hbar^2$. The former is the solution of the equation $\Delta p(T_c) = 0$, and the latter is given by $L = T_c \Delta p'(T_c)$. Unfortunately, the zeroth order term T_0 of the critical temperature $T_c = T_0 + \hbar T_1 + \hbar^2 T_2$ is given as the solution of the equation

$$m_3^2(T_0, \mu) = 0 \ , \tag{18}$$

and the vanishing of m_3^2 leads to singularities at high enough orders. The first order term T_1 is still finite, and given by

$$T_1 = -\frac{\Delta p_1(T_0)}{\Delta p_0'(T_0)} = \frac{3}{4\pi} h_3 m_D \left(\frac{dm_3^2}{dT} \right)^{-1} \Bigg|_{T=T_0} \ . \tag{19}$$

To leading order in the coupling constants, this agrees with the result in ref. [15]. However, the second order term T_2, which includes a finite piece cancelling the μ-dependence of T_0 arising from eq. 18, also includes a logarithmic divergence $T_2 \propto \log(\mu/m_3)$. Hence, T_c is not calculable to order $\hbar^2$. As to the latent heat, both the zeroth order term $L_0 = T_0 \Delta p_0'(T_0)$ and the first order term

$$L_1 = -\Delta p_1(T_0) + T_0 \left(\Delta p_1' - \frac{\Delta p_1 \Delta p_0''}{\Delta p_0'} \right) \Bigg|_{T=T_0} \tag{20}$$

vanish. For L_0 this is natural, but for L_1 it is not, since the 1-loop effective potential $V(\varphi)$ in the Landau gauge has a barrier between the symmetric and broken minima, suggesting a first-order phase transition. The second order term L_2 is finite (after cancellations of various divergences, proportional to m_3^{-2}, m_3^{-1} and $\log m_3$), but it includes an imaginary part:

$$\begin{aligned} L_2 \ = \ & \frac{T_0^2}{16\pi^2} \left\{ -\frac{\lambda_3}{2} \left[\frac{4}{3} + i \left(\frac{g_3^3}{4\lambda_3^{3/2}} + \frac{2^{3/2}}{3} \right) \right]^2 \right. \\ & \left. + \frac{1}{4\lambda_3} \left(\frac{51}{16} g_3^4 + 9\lambda_3 g_3^2 - 12\lambda_3^2 \right) \right\} \left(\frac{dm_3^2}{dT} \right) \Bigg|_{T=T_0} \ . \end{aligned} \tag{21}$$

The real part of L_2 behaves qualitatively like the g^4, λ^2 -curve in Fig. 6 of ref. [2], but the numerical value is much smaller. We conclude that little constructive information concerning the actual EW phase transition can be obtained from the present approach.

Even if the physical properties of the EW phase transition cannot be reliably calculated in powers of $\hbar$, it might be hoped that quantities related solely to the broken phase, away from the critical temperature, could be calculated. The problem is that there may be μ-dependent vacuum parts in these quantities as in the value of pressure. However, when one is comparing perturbation theory with lattice calculations, μ can be fixed [16], and the vacuum parts are not a problem. To illustrate the convergence of

these calculations, we note that for $m_H = 80$ GeV, $\mu = 100$ GeV and $T = 60$ GeV, the $\hbar^0$, $\hbar^1$ and $\hbar^2$ contributions to $\langle \Phi^\dagger \Phi \rangle$ are $[25.47, 3.76$ and $0.39] \times 10^3$ GeV^2 (for these parameters, $T_c \approx 170$ GeV). These values seem to indicate reasonable convergence. In the next Section, the convergence of perturbation theory in the broken phase is studied from a different point of view.

6 Convergence in the broken phase

Above, we were careful to calculate quantities which are gauge-independent to each order in $\hbar$, and we could just hope that this expansion converges. One could also take a different approach, trying to extremize convergence but not being too careful about the gauge dependence. The possible gauge dependence of the physical observables obtained in this way could then be used as a measure of the convergence of the expansion.

Good convergence of perturbation theory requires that the coupling constants, and the higher order contributions including logarithms of the renormalization scale μ, are small. The latter requirement can be satisfied by making a renormalization group (RG) improvement to the naive calculation. In practice, this can be implemented by choosing μ to correspond to a typical mass scale appearing in the propagators. When calculating the effective potential in a large range of φ, the masses in the propagators depend on the shifted field φ, and hence μ should also be chosen to depend on φ. When this is consistently implemented, one arrives at the RG-improved effective potential [3, 17].

In ref. [9], the physical observable $dp/dm^2 = -\langle \Phi^\dagger \Phi \rangle$ was chosen as the indicator of the convergence of the RG-improved loop expansion. The value of $\langle \Phi^\dagger \Phi \rangle$ was calculated numerically from the RG-improved 2-loop effective potential. Due to RG-improvement, and the fact that one is calculating the location of the broken minimum exactly instead of using eq. 9, the result includes a certain subset of contributions from higher powers of $\hbar$. Since $\langle \Phi^\dagger \Phi \rangle$ is gauge-independent in the full theory, the gauge dependence of the mixed-order result tells something about the order of magnitude of the missing terms. It is seen in Fig. 3 of ref. [9] that $\sqrt{2\langle \Phi^\dagger \Phi \rangle}$ depends much less on the gauge fixing parameter than, for instance, the location of the broken minimum, which is not gauge-independent. To give a numerical illustration, for $\xi = 0$ and the parameters cited at the end of Sec. 5Other physical quantities in powers of $\hbar$section.5, the value of $\langle \Phi^\dagger \Phi \rangle$ is 29.64×10^3 GeV^2, and the uncertainty due to gauge dependence is of order 1 percent. This gives support to the argument that the RG-improved loop expansion converges well deep in the broken phase.

7 Conclusions

By calculating consistently in powers of $\hbar$, we have been able to derive a gauge-independent expression for the pressure of EW matter at high temperature. Unfortu-

nately, the physical properties of the EW phase transition cannot be reliably calculated. However, quantities related solely to the broken phase are calculable in powers of $\hbar$, and the convergence of perturbation theory should be reasonably good.

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Appendix

We present here the 2-loop effective potential of the theory defined by eqs. 3 and 4. The tree-level part is

$$V_0(\varphi) = \frac{1}{2} m_3^2 \varphi^2 + \frac{1}{4}\lambda_3 \varphi^4 \ . \tag{22}$$

With the functions $R_\pm$ defined in eq. 6, the 1-loop contribution to the effective potential is

$$V_1(\varphi) = -\frac{1}{12\pi}[6m_T^3 + 3m_L^3 + m_1^3 + 3m_2^3(R_+^3 + R_-^3)] \ . \tag{23}$$

In dimensional regularization, there are no divergences in $V_1(\varphi)$. To present the 2-loop contribution, we use the function

$$H(m_a, m_b, m_c) = \frac{1}{16\pi^2}\left[\frac{1}{4\epsilon} + \log\left(\frac{\mu}{m_a + m_b + m_c}\right) + \frac{1}{2}\right] , \tag{24}$$

arising from the sunset diagrams in Fig. 1. For brevity, we also use the functions $I(m_a, m_b, m_c)$, $D_{VVS}(M, M, m)$ and $D_{SSV}(m_a, m_b, M)$ defined in ref. [3]. These functions are shorthands for certain combinations of masses and logarithms, and the function $H(m_a, m_b, m_c)$ appears linearly in them. Four more functions, denoted by $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$, are defined at the end of this Appendix. A common factor $1/16\pi^2$, and the factor $\mu^{-4\epsilon}$ multiplying $H(m_a, m_b, m_c)$, have been omitted from all the formulas below; accordingly, the factor $1/16\pi^2$ in eq. 24 should also be left out. The divergent part of $V_2(\varphi)$ is obtained from the coefficient of the function $H(m_a, m_b, m_c)$, and is displayed in eq. 7. To get the renormalized expression for $V_2(\varphi)$, the term $1/\epsilon$ is to be omitted from eq. 24, and m_3^2 is to be replaced by $m_3^2(\mu)$. Let us note that due to the identities $R_+^2 + R_-^2 = 1$ and $R_+^2 R_-^2 = \xi(m_T/m_2)^2$, it might be possible to simplify some of the formulas below. We denote $m_2 R_\pm$ by $m_\pm$. Finally, for pure SU(2)-Higgs theory without the A_0^a-field, the graphs (g3), (e2), (g4), (b) and (e1) are left out.

$$\frac{(d2)}{g_3^2} = m_T^2\left(2 + \frac{\xi^{3/2}}{R_+ + R_-}\right)^2$$

$$\frac{(h4)}{3\lambda_3/4} = m_1^2 + 2m_1 m_2 \frac{R_+^5 - R_-^5}{R_+^2 - R_-^2} + 5m_2^2\left(\frac{R_+^5 - R_-^5}{R_+^2 - R_-^2}\right)^2$$

$$\frac{(f2)}{3g_3^2/8} = m_T\left(m_1 + 3m_2\frac{R_+^5 - R_-^5}{R_+^2 - R_-^2}\right)\left(2 + \frac{\xi^{3/2}}{R_+ + R_-}\right)$$

$$\frac{(g3)}{15\lambda_A/4} = m_L^2$$

$$\frac{(e2)}{3g_3^2} = m_L m_T\left(2 + \frac{\xi^{3/2}}{R_+ + R_-}\right)$$

$$
\begin{aligned}
\frac{(g4)}{3h_3/2} &= m_L\left(m_1+3m_2\frac{R_+^5-R_-^5}{R_+^2-R_-^2}\right)\\
\frac{(a)}{-3g_3^4\varphi^2/16} &= D_{VVS}(m_T,m_T,m_1)\\
&\quad+\frac{\xi}{2(m_+^2-m_-^2)}\left\{\frac{R_+^2}{R_-^2}\Big[D_{SSV}(0,m_1,m_T)-D_{SSV}(m_-,m_1,m_T)\Big]-(+\leftrightarrow -)\right\}\\
&\quad+\frac{\xi^2}{(m_+^2-m_-^2)^2}\left\{\frac{R_+^4}{R_-^4}\Big[I(0,0,m_1)-2I(0,m_-,m_1)+I(m_-,m_-,m_1)\Big]\right.\\
&\quad\left.-\Big[I(0,0,m_1)-I(0,m_+,m_1)-I(0,m_-,m_1)+I(m_-,m_+,m_1)\Big]+(+\leftrightarrow -)\right\}\\
\frac{(b)}{-3h_3^2\varphi^2} &= H(m_1,m_L,m_L)\\
\frac{(c)}{-3\lambda_3^2\varphi^2} &= H(m_1,m_1,m_1)\\
&\quad+\frac{1}{(R_+^2-R_-^2)^2}\left[R_+^8H(m_+,m_+,m_1)-R_-^4R_+^4H(m_-,m_+,m_1)+(+\leftrightarrow -)\right]\\
\frac{(e1)}{-3g_3^2/2} &= (m_T^2-4m_L^2)H(m_T,m_L,m_L)+2m_Tm_L-m_L^2\\
&\quad+\frac{4\xi}{m_+^2-m_-^2}\left\{\frac{R_+^2}{R_-^2}\Big[I(0,m_L,m_L)-I(m_-,m_L,m_L)\Big]-(+\leftrightarrow -)\right\}\\
&\quad-\xi m_L^2+\frac{\xi^2m_T^2}{R_+^2-R_-^2}\Big[H(m_-,m_L,m_L)-H(m_+,m_L,m_L)\Big]\\
\frac{(f1)}{-3g_3^2/8} &= \mathcal{A}(m_1,m_2,m_T)+\frac{1}{R_+^2-R_-^2}\Big[R_+^4\mathcal{A}(m_+,m_2,m_T)-R_-^4\mathcal{A}(m_-,m_2,m_T)\Big]\\
\frac{(gh)}{3g_3^2m_T^2/4} &= H(m_T,0,0)+\frac{\xi^2}{R_+^2-R_-^2}\Big[H(m_-,0,0)-H(m_+,0,0)\Big]\\
\frac{(x1)}{3\lambda_3\xi^2m_T^4/(m_+^2-m_-^2)^2} &= (m_+-m_-)^2\\
&\quad+\Big[(2m_-^2-m_1^2)H(m_1,m_-,m_-)-(m_2^2-m_1^2)H(m_1,m_-,m_+)+(+\leftrightarrow -)\Big]\\
\frac{(x2)}{-6\lambda_3\xi m_T^2m_2^2/(m_+^2-m_-^2)^2} &= -R_-^4\Big\{(m_--m_+)(m_--m_1)\\
&\quad+(m_-^2-m_1^2)\Big[H(m_1,m_-,m_-)-H(m_1,m_+,m_-)\Big]\Big\}+(+\leftrightarrow -)\\
\frac{(x3)}{3g_3^2\xi m_T^2/4(m_+^2-m_-^2)} &= D_{SSV}(m_1,m_-,m_T)-D_{SSV}(m_1,m_+,m_T)+\left\{\frac{\xi R_+^2}{R_+^2-R_-^2}\Big[\right.\\
&\quad(m_--m_+)(m_--m_1)+(2m_-^2-m_1^2)H(m_1,m_-,m_-)-(m_2^2-m_1^2)H(m_1,m_+,m_-)\\
&\quad\left.+\frac{4}{m_-^2}\Big[I(m_-,0,m_1)-I(m_-,m_-,m_1)-I(m_+,0,m_1)+I(m_+,m_-,m_1)\Big]\Big]-(+\leftrightarrow -)\right\}
\end{aligned}
$$

$$\frac{(x4)}{-3g_3^2\xi^2 m_T^2/8} = \mathcal{B}(m_1,m_2,m_T) - \frac{2}{R_+^2 - R_-^2}\left[R_+^4\mathcal{B}(m_+,m_2,m_T) - R_-^4\mathcal{B}(m_-,m_2,m_T)\right]$$

$$\begin{aligned}\frac{(x5)}{3g_3^2\xi^2 m_T^2/(m_+^2-m_-^2)^2} &= m_-^4 + m_+^4 - m_- m_+ m_2^2 + \Big\{ I(m_-,m_T,m_-) \\ &-I(m_-,m_T,m_+) + m_T^2 m_-^2\Big[H(m_-,m_+,m_T) - H(m_-,m_-,m_T)\Big] + (+\leftrightarrow -)\Big\}\end{aligned}$$

$$\begin{aligned}\frac{(d1)}{g_3^2} &= \frac{m_T^2}{8}\Big[63H(m_T,m_T,m_T) - 3H(m_T,0,0) - 41\Big] \\ &- \frac{3\xi}{2(R_+^2-R_-^2)}\left[R_+^2\mathcal{C}(m_T,m_-) - R_-^2\mathcal{C}(m_T,m_+)\right] \\ &+ \frac{3\xi^2}{2(R_+^2-R_-^2)^2}\left[R_+^4\mathcal{D}(m_T,m_-,m_-) - R_+^2R_-^2\mathcal{D}(m_T,m_-,m_+) + (+\leftrightarrow -)\right]\end{aligned}$$

The functions $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$ appearing above are

$$\begin{aligned}\mathcal{A}(m_1,m_2,m_T) &= \frac{1}{R_+^2-R_-^2}\left[R_+^4 D_{SSV}(m_1,m_+,m_T) - R_-^4 D_{SSV}(m_1,m_-,m_T)\right] \\ &+\frac{\xi}{(R_+^2-R_-^2)^2}\Big\{-R_+^2R_-^4\Big[2m_-^2 + m_1m_- - D_{SSV}(m_1,m_-,m_-) - (m_-^2+2m_1^2)H(m_1,m_-,m_-)\Big] \\ &+R_+^6\Big[2m_-(m_++m_1) - m_1m_+ - D_{SSV}(m_1,m_+,m_-) \\ &+(m_-^2 - 2m_+^2 - 2m_1^2)H(m_1,m_-,m_+)\Big] + (+\leftrightarrow -)\Big\}\end{aligned}$$

$$\begin{aligned}\mathcal{B}(m_1,m_2,m_T) &= \frac{1}{(m_+^2-m_-^2)^2}\Big\{-m_1^2(m_+-m_-)^2 \\ &+\Big[(m_1^2-m_-^2)^2H(m_1,m_-,m_-) - (m_1^2-m_-^2)(m_1^2-m_+^2)H(m_1,m_-,m_+) + (+\leftrightarrow -)\Big]\Big\}\end{aligned}$$

$$\mathcal{C}(m_T,m) = \frac{1}{2m^2}\left[-m_T^4H(m_T,0,0) + (m_T^2-m^2)^2H(m_T,m,0) + mm_T\left(m_T^2 + \frac{19m^2}{3}\right)\right]$$

$$\begin{aligned}\mathcal{D}(m_T,m_1,m_2) &= \frac{m_T^2}{4m_1^2m_2^2}\Big\{(m_T^2-m_1^2)^2H(m_T,m_1,0) + (m_T^2-m_2^2)^2H(m_T,m_2,0) \\ &-m_T^4H(m_T,0,0) - \Big[m_T^2-(m_1-m_2)^2\Big]\Big[m_T^2-(m_1+m_2)^2\Big]H(m_T,m_1,m_2)\Big\} \\ &+\frac{1}{4m_1m_2}\Big[m_T^3(m_T+m_1+m_2) - m_T^2(m_1^2+m_2^2) - 8m_1^2m_2^2/3\Big]\ .\end{aligned}$$

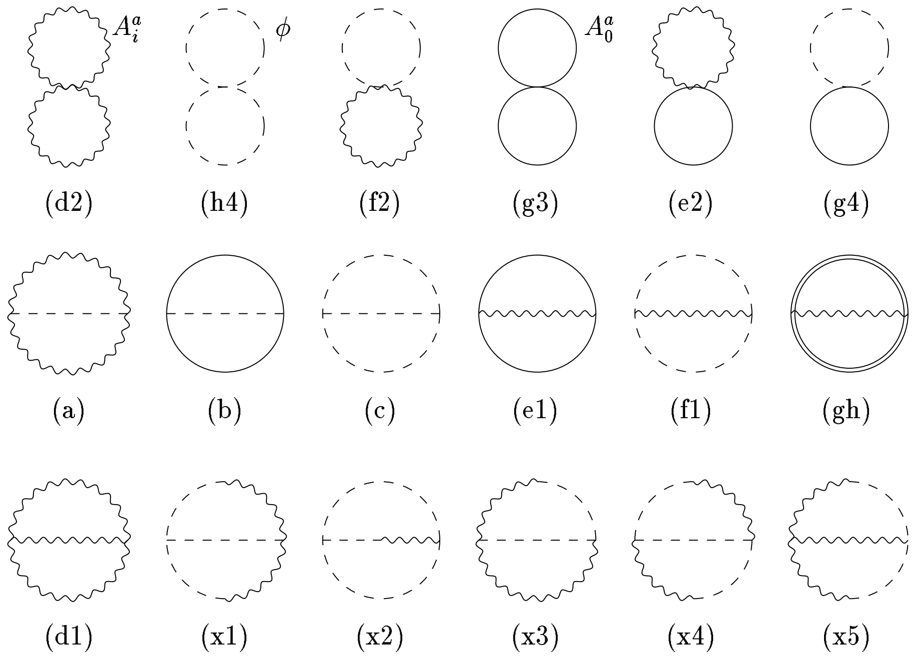

Fig. 1: The non-vanishing 2-loop graphs. Dashed line is the scalar propagator, wiggly line is the A_i^a-propagator, solid line is the A_0^a-propagator, and double line is the ghost propagator. The notations are the same as in ref. [3].

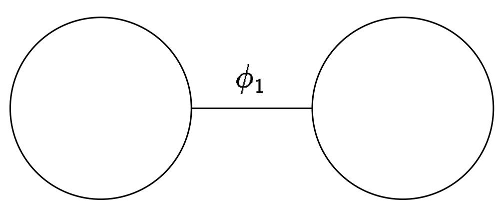

Fig. 2: The general structure of the connected reducible diagrams that are not included in the calculation of the effective potential, but must be included, if one is calculating the partition function in the loop expansion without using the effective potential. Only Higgs particles can propagate on the straight line.